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Neutrino Mixing and Geometric CP Violation with $\Delta(27)$ Symmetry

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Abstract

Predictive spontaneous CP violation is possible if it is obtained geometrically through a non-Abelian discrete symmetry. I propose such a model of neutrino mass and mixing based on $\Delta(27)$.

Since the experimental determination of nonzero θ_{13} in neutrino oscillations, the next big question in neutrino physics is CP violation. Theoretically, this should be understood together with the mixing angles themselves. Whereas non-Abelian discrete symmetries (the first [1, 2, 3, 4] of which was A_4) are useful in obtaining tribimaximal mixing [5] which requires $\theta_{13} = 0$ and no CP violation, the data now require either a modification or a new approach. In the former, CP violation may be incorporated by allowing nonzero θ_{13} and complex Yukawa couplings. A simple example is a variation [6] of the original A_4 model [4] for tribimaximal mixing. In the latter, the discrete symmetry may be extended to include generalized CP transformations [7], which in the case [8] of S_4 could lead to maximal CP violation as well as maximal θ_{23}. Another possible approach in this category is spontaneous geometric CP violation [9] using $\Delta(27)$, which has recently been applied [10] successfully to the quark sector. This paper deals with the lepton sector [11, 12, 13] and how it may be related [14] to dark matter.

The non-Abelian discrete symmetry $\Delta(27)$ has 27 elements, with nine one-dimensional irreducible representations $\underline{1}_i$ $(i = 1, ..., 9)$ and two three-dimensional ones $\underline{3}$ and $\underline{3}^*$. Its 11×11 character table as well as the 27 defining 3×3 matrices of its $\underline{3}$ representation are given in Ref. [11]. The group multiplcation rules are

$$\underline{3} \times \underline{3} = \underline{3}^* + \underline{3}^* + \underline{3}^*, \qquad \underline{3} \times \underline{3}^* = \sum_{i=1}^{9} \underline{1}_i. \tag{1}$$

The important property to notice is that $\underline{3} \times \underline{3} \times \underline{3}$ has three invariants: $123 + 231 + 312 - 213 - 321 - 132$ [which is also invariant under $SU(3)$], $123 + 231 + 312 + 213 + 321 + 132$ [which is also invariant under A_4], and $111 + 222 + 333$.

In this paper, the assignments of the lepton and Higgs fields are different from previous studies [11, 12, 13], with the new requirement that CP be spontaneously broken geometri-

cally [9, 10]. Let

$$\begin{pmatrix} \nu \\ l \end{pmatrix}_i \sim \underline{3}, \quad l_i^c \sim \underline{1}_1, \underline{1}_2, \underline{1}_2, \quad \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}_i \sim \underline{3}. \tag{2}$$

Using the decomposition of $\underline{3} \times \underline{3}^*$ and $\langle \phi_i^0 \rangle = v_i$, the charged-lepton mass matrix is given by

$$\mathcal{M}_l = \begin{pmatrix} f_e v_1^* & f_\mu v_1^* & f_\tau v_1^* \\ f_e v_2^* & f_\mu \omega^2 v_2^* & f_\tau \omega v_2^* \\ f_e v_3^* & f_\mu \omega v_3^* & f_\tau \omega^2 v_3^* \end{pmatrix} = \begin{pmatrix} v_1^* & 0 & 0 \\ 0 & v_2^* & 0 \\ 0 & 0 & v_3^* \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \begin{pmatrix} f_e & 0 & 0 \\ 0 & f_\mu & 0 \\ 0 & 0 & f_\tau \end{pmatrix}, \tag{3}$$

where $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$. This $\mathcal{M}_l$ is identical in form to that of the original A_4 model of Ref. [1]. The new feature here is that CP conservation is imposed on the Lagrangian (so that all the Yukawa couplings are real) but it is spontaneously broken by the vacuum, i.e. [9, 10]

$$(v_1, v_2, v_3) = v(\omega, 1, 1). \tag{4}$$

Hence

$$\mathcal{M}_l = \begin{pmatrix} \omega^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \tag{5}$$

where $m_e = \sqrt{3} f_e v$, etc.

For the neutrino mass matrix, three Higgs doublets

$$\begin{pmatrix} \zeta^+ \\ \zeta^0 \end{pmatrix}_i \sim \underline{1}_1, \underline{1}_2, \underline{1}_3 \tag{6}$$

are added so that the dimension-five operator $\Lambda^{-1}(\nu\nu\phi^0)\zeta^0$ for the 3×3 Majorana neutrino mass matrix has six invariants, i.e.

$$\mathcal{M}_\nu = \begin{pmatrix} \omega(f_1 + f_2 + f_3) & f_4 + \omega f_5 + \omega^2 f_6 & f_4 + \omega^2 f_5 + \omega f_6 \\ f_4 + \omega f_5 + \omega^2 f_6 & f_1 + \omega^2 f_2 + \omega f_3 & \omega(f_4 + f_5 + f_6) \\ f_4 + \omega^2 f_5 + \omega f_6 & \omega(f_4 + f_5 + f_6) & f_1 + \omega f_2 + \omega^2 f_3 \end{pmatrix}, \tag{7}$$

where $\Lambda^{-1} v \langle \zeta_i^0 \rangle$ have been absorbed into the definitions of the f parameters.

Using Eq. (5), the neutrino mass matrix in the tribimaximal basis is now given by

$$\mathcal{M}_\nu^{(1,2,3)} = \begin{pmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ \omega & 0 & 0 \\ 0 & -i/\sqrt{2} & i/\sqrt{2} \end{pmatrix} \mathcal{M}_\nu \begin{pmatrix} 0 & \omega & 0 \\ 1/\sqrt{2} & 0 & -i/\sqrt{2} \\ 1/\sqrt{2} & 0 & i/\sqrt{2} \end{pmatrix}$$

$$= \begin{pmatrix} \omega d + b & \omega e & c \\ \omega e & a & f \\ c & f & \omega d - b \end{pmatrix}, \tag{8}$$

where $a = f_1 + f_2 + f_3$, $b = f_1 - (f_2 + f_3)/2$, $c = \sqrt{3}(f_3 - f_2)/2$, $d = f_4 + f_5 + f_6$, $e = \sqrt{2} f_4 - (f_5 + f_6)/\sqrt{2}$, $f = \sqrt{3}(f_5 - f_6)/\sqrt{2}$. The tribimaximal limit, i.e.

$$U_{l\nu} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{6} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \tag{9}$$

is reached for $c = e = f = 0$. To lowest order, $c \neq 0$ implies $\tan^2\theta_{12} > 0.5$ and $\theta_{13} \neq 0$; $e \neq 0$ implies $\tan^2\theta_{12}$ can be greater or less than $1/2$ and $\theta_{13} = 0$; $f \neq 0$ implies $\tan^2\theta_{12} < 1/2$ and $\theta_{13} \neq 0$. Given that data prefer the last choice, it will be assumed from now on that c and e are negligible and only nonzero f is considered. The immediate consequence [6] of this is that θ_{12} and θ_{13} are related, and that given θ_{13} and θ_{23}, $|\tan\delta_{CP}|$ is determined.

Since $c = e = 0$ has been assumed, $\mathcal{M}_\nu^{(1,2,3)}$ is diagonalized by

$$\begin{pmatrix} m_2 & 0 \\ 0 & m_3 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta e^{i\phi} \\ -\sin\theta e^{-i\phi} & \cos\theta \end{pmatrix} \begin{pmatrix} a & f \\ f & \omega d - b \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta e^{-i\phi} \\ \sin\theta e^{i\phi} & \cos\theta \end{pmatrix}. \tag{10}$$

Since a, b, d, f are real, this implies

$$\tan\phi = \frac{\sqrt{3}d/2}{b - a + d/2}, \qquad \tan 2\theta = \frac{f\sqrt{(b-a)^2 + d(b-a) + d^2}}{b^2 - a^2 + db + d^2}. \tag{11}$$

With this structure, $|\sin\theta_{13}| = |\sin\theta|/\sqrt{3}$, which implies

$$\tan^2\theta_{12} = \frac{1 - 3\sin^2\theta_{13}}{2}, \tag{12}$$

which agrees very well [6] with data. As for the phase ϕ, it is given by the condition

$$\tan^2\theta_{23} = \frac{\left(1 - \frac{\sqrt{2}\sin\theta_{13}\cos\phi}{\sqrt{1-3\sin^2\theta_{13}}}\right)^2 + \frac{2\sin^2\theta_{13}\sin^2\phi}{1-3\sin^2\theta_{13}}}{\left(1 + \frac{\sqrt{2}\sin\theta_{13}\cos\phi}{\sqrt{1-3\sin^2\theta_{13}}}\right)^2 + \frac{2\sin^2\theta_{13}\sin^2\phi}{1-3\sin^2\theta_{13}}}. \tag{13}$$

Since m_2^2 and m_3^2 are corrected by terms proportional to f^2 which are small, the following approximation for the neutrino masses is valid for the analysis below, i.e.

$$m_1 = \sqrt{b^2 - db + d^2}, \quad m_2 = |a|, \quad m_3 = \sqrt{b^2 + db + d^2}. \tag{14}$$

Hence $2bd = \pm|\Delta m_{32}^2| \equiv \pm\Delta$ for normal (inverted) ordering of neutrino masses. Since $\Delta m_{21}^2 << \Delta$, $m_1 \simeq m_2$ will be also assumed below.

Let $\Delta = 2.35 \times 10^{-3}$ eV, which is the central value from the 2012 PDG compilation, then using $d = \pm\Delta/2b$ and $a = \pm\sqrt{b^2 - bd + d^2}$, this model has the prediction

$$\sum m > (2 + \sqrt{3})\sqrt{\frac{\Delta}{2}} = 0.13 \text{ eV for normal ordering,} \tag{15}$$

$$\sum m > (2\sqrt{3} + 1)\sqrt{\frac{\Delta}{2}} = 0.15 \text{ eV for inverted ordering.} \tag{16}$$

Using the latest Planck result [15] that $\sum m < 0.23$ eV, the range of values for b is also obtained:

$$0.015 < b < 0.078 \text{ eV for normal ordering,} \tag{17}$$

$$0.016 < b < 0.073 \text{ eV for inverted ordering.} \tag{18}$$

Using Eq. (13) for $\sin^2 2\theta_{23} > 0.92$ and $\sin^2 2\theta_{13} \simeq 0.1$, the constraint

$$|\tan\phi| > 1, \quad \text{or} \quad |\sin\phi| > 1/\sqrt{2} \tag{19}$$

is obtained. Since b and ϕ are related by Eq. (11), the range of allowed values for b is further restricted to

$$0.015 < b < 0.060 \text{ eV,} \tag{20}$$

and holds for normal ordering only with $a > 0$. There is no solution for inverted ordering with $|\tan\phi| > 1$. The invariant CP violating parameter $J_{CP} = Im(U_{\mu 3}U_{e3}^* U_{e2} U_{\mu 2}^*)$ is simply given in this model by

$$J_{CP} = \frac{\sin\theta_{13}\sqrt{1 - 3\sin^2\theta_{13}}\sin\phi}{3\sqrt{2}}. \tag{21}$$

Using $\sin\theta_{13} \simeq 0.16$ and $|\sin\phi| > 1/\sqrt{2}$, the allowed range

$$0.026 < |J_{CP}| < 0.036 \tag{22}$$

is thus obtained. Using Eq. (20), the allowed range for the effective neutrino mass in neutrinoless double beta decay is approximately given by

$$0.03 < m_{ee} < 0.05 \text{ eV}. \tag{23}$$

Thus this model has very specific predictions: (1) only normal ordering of neutrino masses is allowed, (2) $|J_{CP}|$ is between 0.026 and 0.036, and (3) m_{ee} is between 0.03 and 0.05 eV.

The dimension-five operator [16] for Majorana neutrino mass considered in the above may be implemented [14] in one loop, with dark matter (Z_2 odd) in the loop. This mechanism

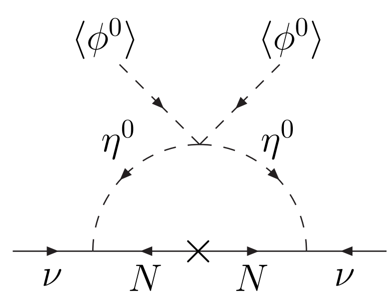

Figure 1: One-loop generation of scotogenic Majorana neutrino mass.

has been called "scotogenic", from the Greek "scotos" meaning darkness. Because of the allowed $(\lambda_5/2)(\Phi^\dagger\eta)^2 + H.c.$ interaction, $\eta^0 = (\eta_R + i\eta_I)/\sqrt{2}$ is split so that $m_R \neq m_I$. The diagram of Fig. 1 can be computed exactly [14], i.e.

$$(\mathcal{M}_\nu)_{ij} = \sum_k \frac{h_{ik}h_{jk}M_k}{16\pi^2}\left[\frac{m_R^2}{m_R^2 - M_k^2}\ln\frac{m_R^2}{M_k^2} - \frac{m_I^2}{m_I^2 - M_k^2}\ln\frac{m_I^2}{M_k^2}\right]. \tag{24}$$

A good dark-matter candidate is η_R as first pointed out in Ref. [14]. It was subsequently proposed by itself in Ref. [18] (to render the standard-model Higgs boson very heavy, which

is now ruled out by data) and studied in detail in Ref. [19]. The η doublet has become known as the "inert" Higgs doublet, but it does have gauge and scalar interactions even if it is the sole addition to the standard model.

To accommodate the $\Delta(27)$ symmetry, the external $\phi^0\phi^0$ lines are replaced by $\phi^0_i\zeta^0_j$, and the internal η^0 (N) lines are replaced by η^0_i, $N_i \sim \underline{3}$ on one side, and $\eta^0 \sim \underline{1}$, $N_i \sim \underline{3}^*$ on the other.

In conclusion, a special mechanism of CP violation has been implemented in a complete model of charged-lepton and neutrino masses and mixing, using the non-Abelian discrete symmetry $\Delta(27)$. The Lagrangian is required to conserve CP resulting in real Yukawa couplings, but the Higgs vacuum breaks CP spontaneously and geometrically. The resulting model has some very specific predictions, as given by Eqs. (12) to (23).

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